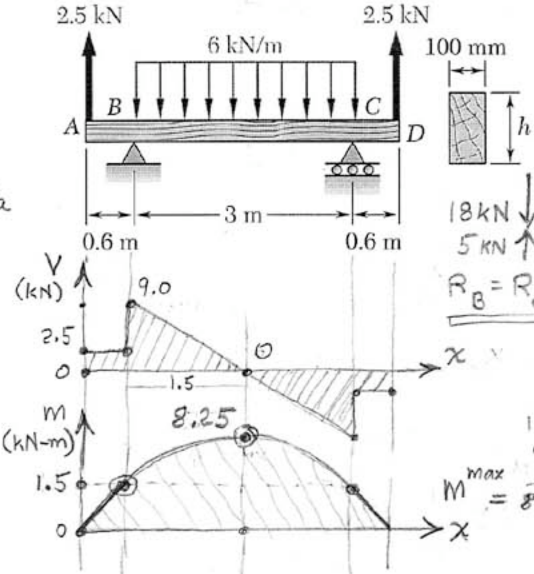
MULTIPLE CHOICE PROBLEMS (10 Points each)

1. The beam shown is made from a wood for which the allowable normal stress is 12 MPa. The minimum required depth of the beam, h, is most nearly



(a) 41.3 mm
(b) 100 mm
(c) 141.3 mm
(d) 203 mm
(e) 212 mm

$$I = \frac{bh^3}{12} = \frac{0.1\,h^3}{12} = \frac{h^3}{120}$$

$$\sigma = \frac{My}{I}$$

$$\sigma^{max} = \frac{M^{max}(h/2)}{h^3/120} = 12\times10^6\ \text{Pa}$$

$$60\,M^{max} = 12\times10^6\,h^2$$

$$60(8.25\times10^3) = 12\times10^6\,h^2$$

$$h^2 = 5(8.25\times10^{-3})\ \text{m}^2$$

$$h^2 = 41.25\times10^{-3}\ \text{m}^2$$

$$\boxed{h = 203\ \text{mm}}$$

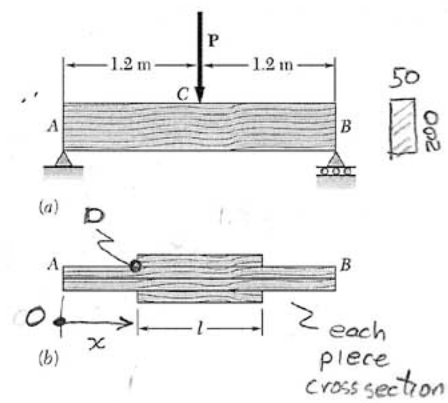
18 kN ↓, 5 kN ↑ } 13 kN ↓

$R_B = R_C = 6.5$ kN ↑

1.5
9
13.5/2
6.75
1.50

$M^{max} = 8.25$ kN-m

2. A preliminary design on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. The minimum length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design is most nearly



each piece cross section is 50 × 50

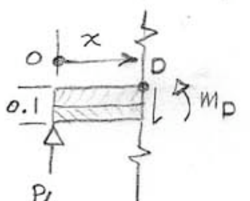
(a) 0.600 m
(b) 1.800 m
(c) 2.40 m
(d) 0.300 m
(e) 0.1500 m

$$F.S.^{(a)} = F.S.^{(b)}$$

$$\frac{\cancel{\sigma_{ult}}}{\sigma^{(a)}_{allow}} = \frac{\cancel{\sigma_{ult}}}{\sigma^{(b)}_{allow}} \quad ; \quad \sigma^{(a)}_{allow} = \sigma^{(b)}_{allow}$$

$$\sigma^{(a)}_{allow} = \sigma^{"C"}_{allow} = \frac{M^{max}\,y}{I} = \frac{[P/2\,(1.2)]\,0.1}{0.05(0.2)^3/12}$$

$$\sigma^{(b)}_{allow} = \sigma^{"D"}_{allow} \Rightarrow \frac{(P/2\,(x))\,0.05}{\cancel{0.05}(0.1)^3/\cancel{12}} = \frac{(P/2\,(1.2))\,0.1}{\cancel{0.05}(0.2)^3/\cancel{12}}$$



$$l/2 + x = 1.2$$

$$l = 2(1.2 - x)$$

$$\frac{0.05\,x}{(0.1)^3} = \frac{0.12}{(0.2)^3}$$

$$x = \frac{0.12}{0.05}\,\frac{(0.1)^3}{(0.2)^3} = \frac{0.12(0.001)}{0.05(0.008)} = 0.3^{m} = x$$

$$l = 2(1.2 - 0.3) = 2(0.9)$$

$$\boxed{l = 1.8\text{m}}$$

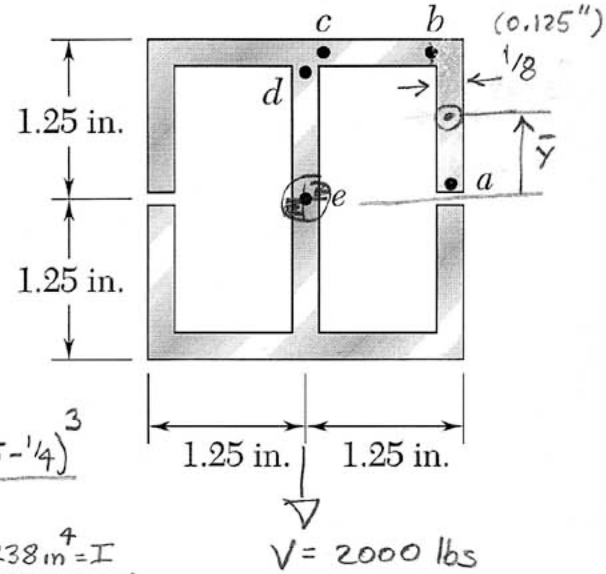
3. The extruded beam shown has a uniform wall thickness of 1/8 in. If the vertical shear in the beam is 2 kips, the shearing stress at point b is most nearly

(a) 0 ksi
(b) 1.262 ksi
(c) 2.04 ksi
(d) 6.40 ksi
(e) 1.362 ksi

$\bar{y} = 0.625''$

$Q = \bar{y}A \qquad A = 0.15625''$

$Q = 0.09766 \text{ in}^3$



$$\tau_b = \frac{VQ}{It} = \frac{2000(0.09766)}{1.238(0.125)} = 1262 \text{ psi}$$

$\boxed{\tau_b = 1.262 \text{ Ksi}}$

$$I = \left.\frac{bh^3}{12}\right|_{outer} - \left.\frac{bh^3}{12}\right|_{inner} = \frac{2.5(2.5)^3}{12} - \frac{(2.5-3/8)(2.5-1/4)^3}{12}$$

(2.125)

$= 3.255 - 2.017 = 1.238 \text{ in}^4 = I$

4. Three boards are nailed together to form the beam shown which is subjected to a vertical shear. If the spacing between the nails is $s = 75$ mm and the allowable shearing force in each nail is 400 N, the allowable shear is most nearly

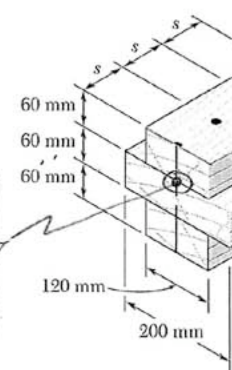
Solve for V

(a) 738 N
(b) 5.33 N
(c) 1023 N
(d) 327 N
(e) 35.6 N

$$F_{nail} = s \cdot q = \frac{\overset{?}{V}Q}{I} \cdot s = 400 \text{ N}$$

$$V = \frac{FI}{Qs} = \frac{400 \text{ N}(59.76 \times 10^{-6}) \text{m}^4}{0.432 \times 10^{-3} \text{m}^3 (0.075) \text{m}}$$

q
F_{nail}
τ

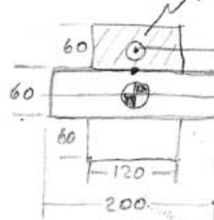


$\boxed{V = 737.7 \text{ N}}$

$Q = \bar{y}A$

$Q = 0.000432 \text{ m}^3$

$A = 0.0072 \text{m}^2$



$\bar{y} = 0.06$ m

$10^{12} \text{ mm}^4 = 1 \text{m}^4$

$$I = \left.\frac{bh^3}{12}\right|_{inner} + 2\left.\left[\frac{bh^3}{12} + Ad^2\right]\right|_{outer}$$

$$I = \left\{\frac{200(60)^3}{12} + 2 \cdot \left[\frac{120(60)^3}{12} + ((60)(120))(60)^2\right]\right\}$$

$$= \left\{3.6 + 2 \cdot [2.16 + 25.92]\right\} \times 10^6 \text{ mm}^4$$

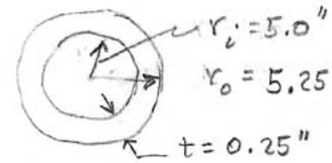
$$\left\{3.6 + 56.16\right\} \times 10^6 \text{ mm}^4 \qquad I = 59.76 \times 10^{-6} \text{ m}^4$$

Check

$$I = \left\{\frac{200(180)^3}{12} - 4\left[\frac{40(60)^3}{12} + ((40)(60))(60)^2\right]\right\}$$

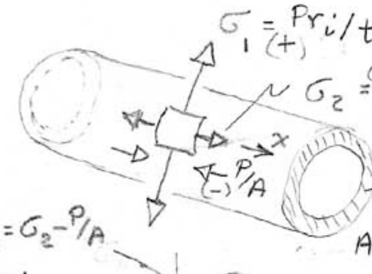
$$= \left\{97.2 - 4\,\underbrace{[0.72 + 8.64]}_{9.36}\right\} \times 10^6 = \left\{97.2 - 37.44\right\} \times 10^6 = 59.76 \times 10^6 \text{ mm}^4 ✓$$

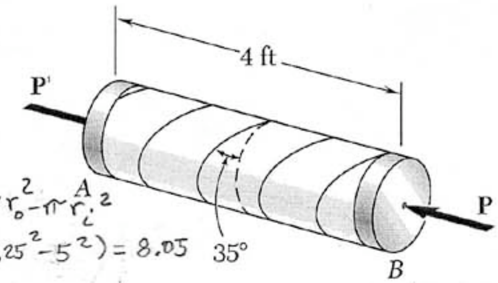
Note: text book use
$r = r_i$ in $\sigma_1 = Pr/t$



5. A pressure vessel of 10-in. inner diameter and 0.25-in. wall thickness is fabricated from a 4-ft section of spirally welded pipe *AB* and is equipped with two rigid end plates. The gage pressure inside the vessel is 300 psi and 10-kip centric forces **P** and **P`** are applied to the end plates. The normal stress perpendicular to the weld is most nearly

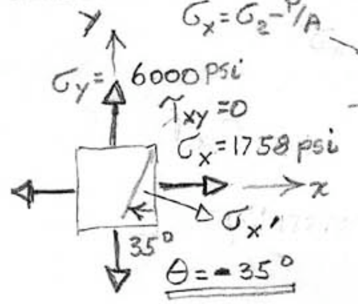
(a) 1.757 ksi
(b) 6.00 ksi
(c) 3.00 ksi
(d) 4.60 ksi
(e) 3.15 ksi





$A = \pi r_o^2 - \pi r_i^2$

$A = \pi(5.25^2 - 5^2) = 8.05$



$\sigma = -P/A = -\dfrac{10{,}000}{8.05} = -1242$ psi

$\sigma_y = \sigma_1 = Pr_i/t = \dfrac{300 \cdot 5}{1/4} = 6000$ psi, σ_y

$\sigma_2 = +3000$ psi

$\sigma_x = 3000 - 1242 = 1758$ psi, σ_x

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}^{\nearrow 0}\sin 2\theta = 3879 - 2121\underbrace{\overset{(0.342)}{(\cos(-70°))}}_{-725.4} = \boxed{3154 \text{ psi} = \sigma_{x'}}$$

6. Square plates, each of 16-mm thickness can be bent and welded together to form the cylindrical portion of a compressed air tank as shown. If the largest allowable normal stress perpendicular to the weld is 65 MPa, the largest allowable gage pressure is most nearly

$r_i = d/2 - t = 5/2 - 0.016$

(a) 0.555 MPa
(b) 0.416 MPa
(c) 0.832 MPa
(d) 0.208 MPa
(e) 0.312 MPa

$\sigma_1 = Pr_i/t$

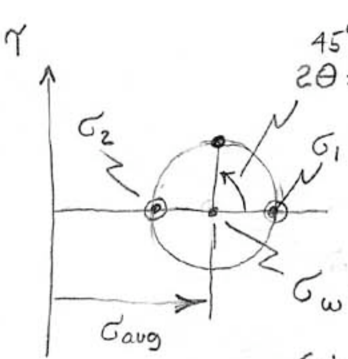
$\sigma_2 = \sigma_1/2 = \dfrac{Pr_i}{2t}$

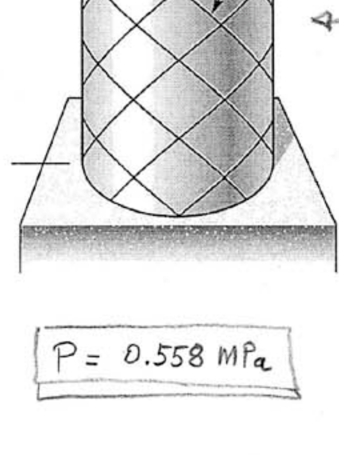
$\sigma_{avg} = \dfrac{3}{4}\, Pr_i/t$

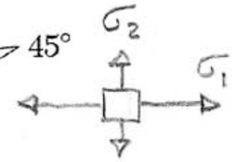
$r_i = 2.484$ m

$t = 0.016$ m

$P = ?$ Find







$\sigma_w = \sigma_{avg} = \dfrac{3}{4}\dfrac{Pr_i}{t} = 65 \times 10^6$ Pa

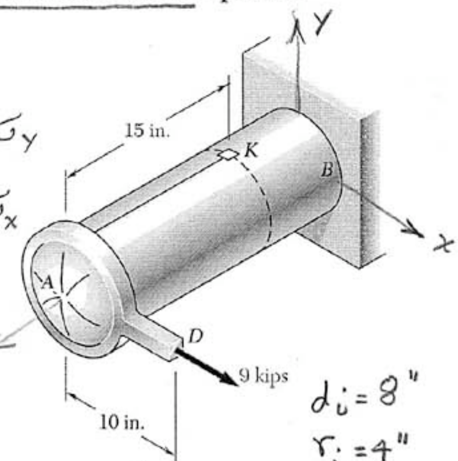
Solve for P

$$P = \frac{4(65\times10^6)\,t}{3\,r_i} = \frac{4(65\times10^6)\,0.016}{3\,(2.484)}$$

$\boxed{P = 0.558 \text{ MPa}}$

WORK OUT PROBLEM (40 Points) **Name:** ____________________

The cylindrical tank *AB* has an 8-in. inner diameter and a 0.32-in. wall thickness. If the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shear stress a point *K* located on the top of the tank.



$\sigma_1 = \frac{P r_i}{t} = \frac{600(4)}{0.32}$

$\sigma_1 = 7500\ psi = \sigma_y$

$\sigma_2 = 3750\ psi = \sigma_x$

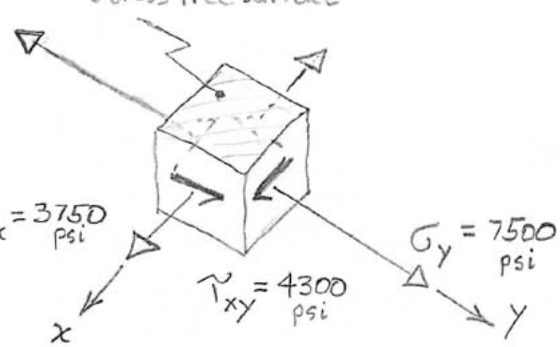
$d_i = 8''$

$r_i = 4''$

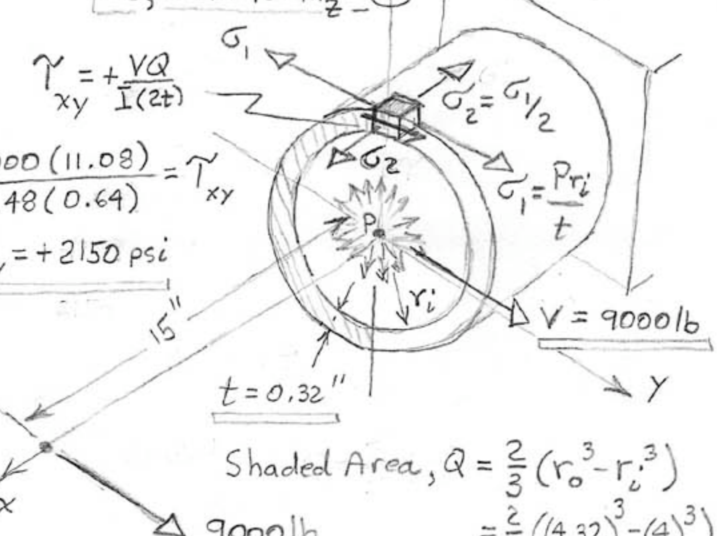
$t = 0.32''$

$r_o = 4.32''$

Plane Stress: $\sigma_z = \tau_{xz} = \tau_{yz} = 0$
stress free surface



Not Needed
$15 \cdot 9000 = M_z$
$135{,}000\ in\text{-}lb = M_z$



$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 5625\ psi = \sigma_{avg}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ $R = 2853\ psi$

$= \sqrt{(1875)^2 + (2150)^2}$

$\sqrt{3.515 \times 10^6 + 4.623 \times 10^6}$

$\sqrt{8.138 \times 10^6}$

$\tau_{xy} = +\frac{VQ}{I(2t)}$

$\frac{9000(11.08)}{72.48(0.64)} = \tau_{xy}$

$\tau_{xy} = +2150\ psi$

$\sigma_{max} = \sigma_{avg} + R$ $\boxed{\sigma_{max} = 8.478\ ksi}$

$\sigma_{min} = \sigma_{avg} - R$ $\sigma_{min} = 2.772\ ksi$

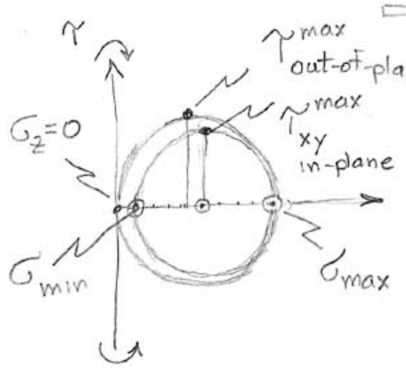
Shaded Area, $Q = \frac{2}{3}(r_o^3 - r_i^3)$

$= \frac{2}{3}\left((4.32)^3 - (4)^3\right)$

$2(80.62 - 64)/3$

$Q = 11.08\ in^3$

$\tau^{max}_{out\text{-}of\text{-}plane} = \frac{1}{2}\left|\sigma_{max} - 0\right|$



$\boxed{\tau^{max}_{out\text{-}of\text{-}plane} = 4.24\ ksi}$

$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}\left((4.32)^4 - (4)^4\right)$

$\pi(348.29 - 256)/4$

$I = 72.48\ in^4$